A Mixed Integer Programming Model for Timed Deliveries in Multirobot Systems

Nitin Kamra and Nora Ayanian
Department of Computer Science
University of Southern California
Los Angeles, CA, USA

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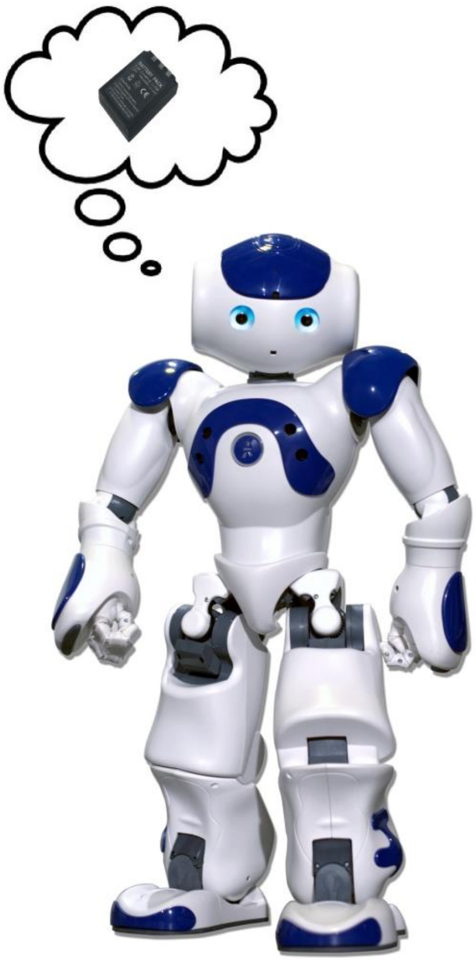
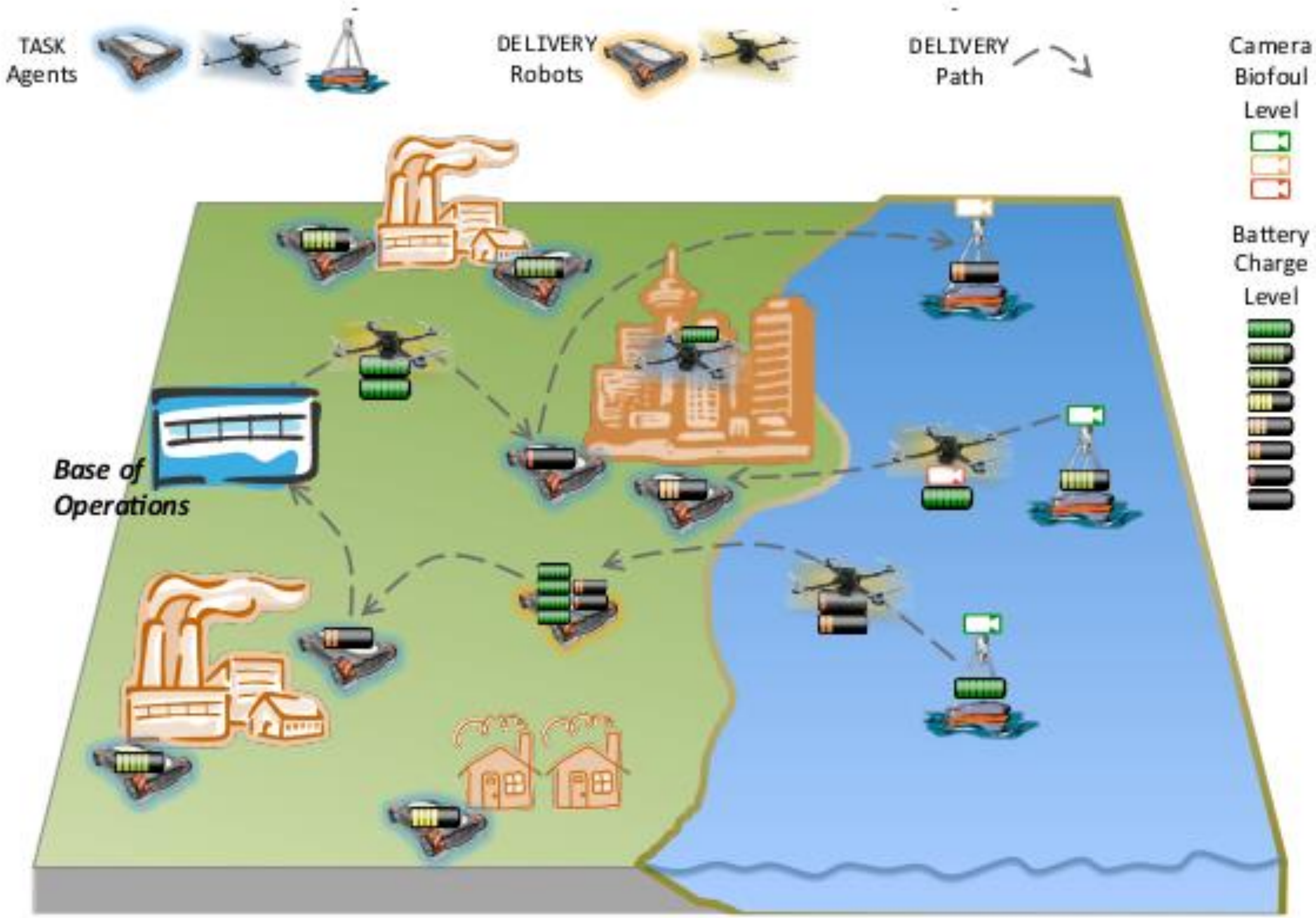
Long Duration Autonomy in Robotics

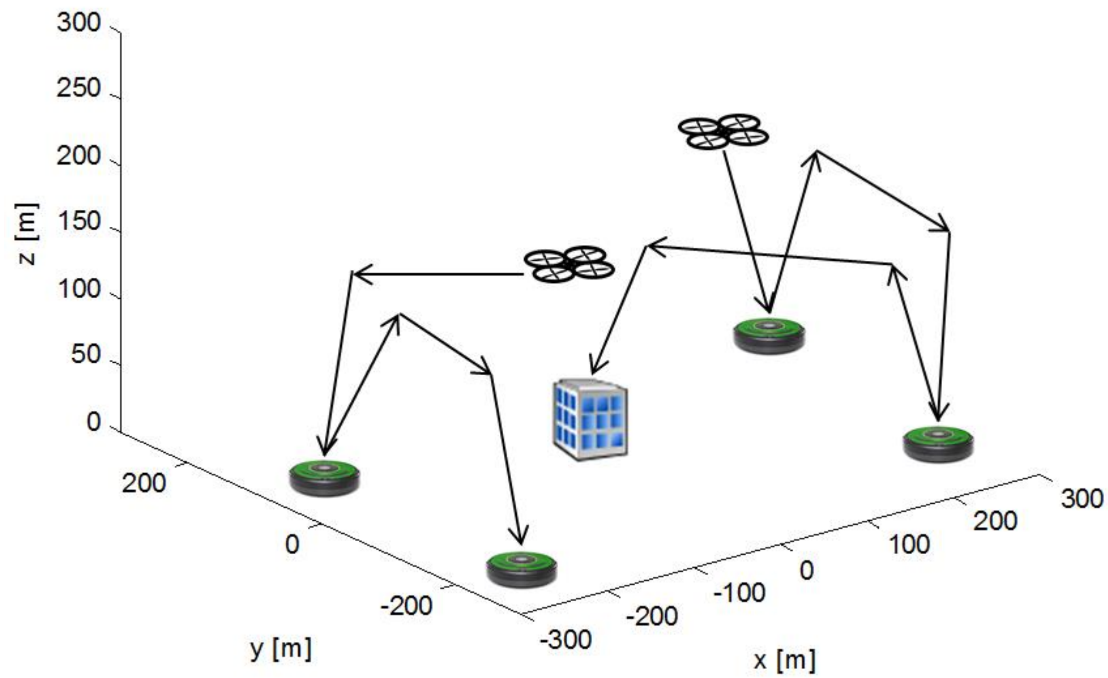


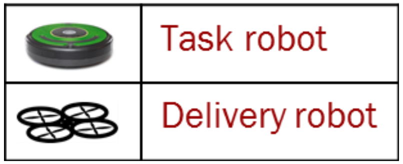
Robots need resources (power, sensors, actuators) for persistent autonomy.

Objective


TASK
Agents
DELIVERY
Robots
DELIVERY
Path
Camera
Biofoul
Level
Battery
Charge
Level
Base of
Operations

Objective





Specifications:

- Delivery robots: Limited on-board power and carrying capacity.
- Priorities over task robots.

Goal:

- Solve the resource delivery problem with timed requests, optimally.
- Minimize total distance traveled and deviation from delivery times.

Existing Approaches to Persistent Autonomy

SOFTWARE-BASED

Persistent Autonomy

Kim & Morrison, JIRS, 2014
(Persistent operation scheduling for UAVs)

Song *et al.* ICUAS, 2013
(Scheduling for persistent UAV service)

Smith *et al.* ICRA, 2011
(Monitoring in dynamic environments)

Energy-aware systems

Derenick *et al.* IROS, 2011
(Energy aware coverage with Docking)

Kannan *et al.* ICRA, 2013
(Autonomous recharging, market-based soln.)

Mathew *et al.* ICRA, 2013
(Multi-robot rendezvous for recharging)

HARDWARE-BASED

Swappable Batteries

Suzuki *et al.* JIRS, 2012
(Design and analysis of battery swapping system)

Swieringa *et al.* ICRA 2010
(Automatic battery swapping for UAVs)

Modeling Paradigms: Delivery Problems

Stochastic Modeling with Queueing Theory:

Bopardikar *et al.* T-RO, 2014
(Dynamic VRP with time constraints)

Smith *et al.* CDC, 2008
(Dynamic VRP with heterogeneous demands)

Smith *et al.* SIAM J Control Optim, 2010
(Dynamic VRP, priority classes of demands)

Mixed-integer based formulations:

Karaman & Frazzoli, IJ Robust Nonlin, 2011
(LTL Vehicle routing; appl. to multi-uav mission planning)

Mathew *et al.* WAFR, 2014
(Path planning, multi-robot delivery systems)

Stump & Michael, IEEE CASE, 2011
(Multi-robot persistent surveillance as vehicle routing)

Impose probability distributions on arrival rates and locations of requests.

Stochastic analysis of policies to serve stochastic requests.

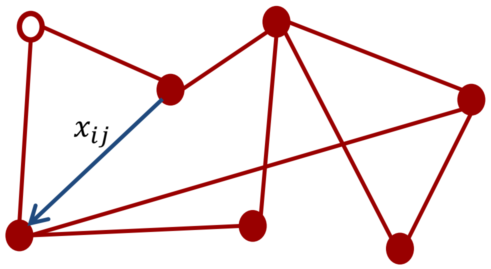
Express the objective algebraically.

Guarantees of optimality.

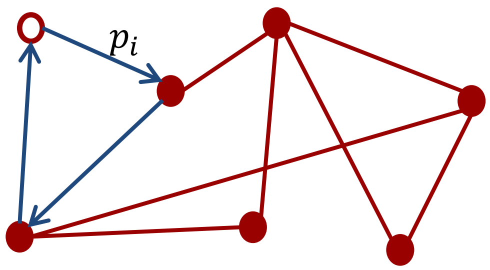
Easier to impose constraints.

Close to VRPTW Formulation.

MIP: Arc vs. Path based approach



Arc-based approach
O(m·n²) variables



Path-based approach
O(m·n!) variables

act lab

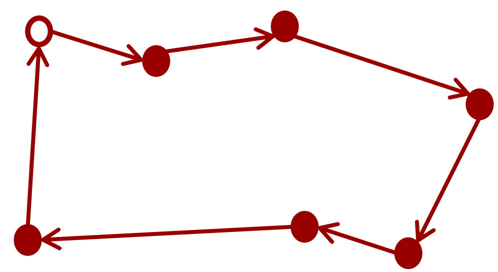
Traveling Salesman Problem - ILP Formulation

$$\min_x \sum_{i \neq j} c_{ij} x_{ij}$$

subject to the constraints:

$$\sum_{j \in V - \{i\}} x_{ij} = 1, \quad \forall i \in V$$	Exit each node once
$$\sum_{i \in V - \{j\}} x_{ij} = 1, \quad \forall j \in V$$	Enter each node once
$$\sum_{i \in S, j \in S} x_{ij} \leq \lvert S \rvert - 1, \quad \forall S \subset V, 2 \leq \lvert S \rvert \leq \lvert V \rvert - 2$$	Eliminate Subtours

$$x_{ij} \in \{0, 1\}, \quad \forall i, j \in V$$



Vehicle Routing Problem - ILP Formulation

$$\min_x \sum_{k \in K} \sum_{(i,j) \in E} x_{ij}^k t_{ij}$$

subject to the constraints:

$$\sum_{k \in K} \sum_{j \in V \cup \{\omega\}} x_{ij}^k = 1, \qquad \forall i \in V$$

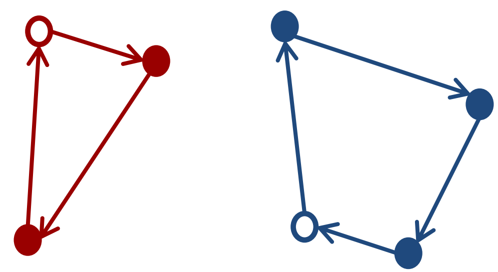
$$x_{\alpha k}^k = 1, \qquad \forall k \in K$$

$$\sum_{i \in V} x_{i\omega}^k = 1, \qquad \forall k \in K$$

"Start" and "End" nodes are visited by all robots

$$\sum_{i \in \{\alpha\} \cup V} x_{ih}^k = \sum_{j \in (V-K) \cup \{\omega\}} x_{hj}^k, \qquad \forall h \in V, k \in K$$

Subtour Elimination Constraints



Vehicle Routing Problem with Capacity Constraints

$$\min_x \sum_{k\in K} \sum_{(i,j)\in E} x_{ij}^k t_{ij}$$

subject to the constraints:

$$\sum_{k\in K} \sum_{j\in V\cup\{\omega\}} x_{ij}^k = 1, \qquad \forall i \in V$$

$$x_{\alpha k}^k = 1, \qquad \forall k \in K$$

$$\sum_{i\in V} x_{i\omega}^k = 1, \qquad \forall k \in K$$

$$\sum_{i\in\{\alpha\}\cup V} x_{ih}^k = \sum_{j\in(V-K)\cup\{\omega\}} x_{hj}^k, \qquad \forall h \in V, k \in K$$

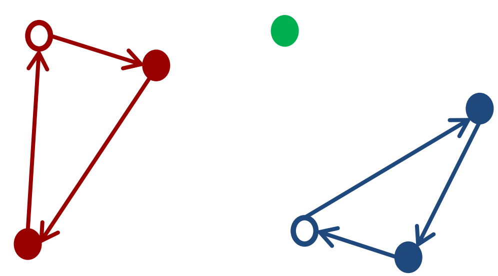
Subtour Elimination Constraints

$$\sum_{(i,j)\in E} x_{ij}^k - 1 \le C^k - c^k, \qquad \forall k \in K$$

Delivery Capacity

$$\sum_{(i,j)\in E} x_{ij}^k B_r^k(t_{ij}v) \le B^k, \qquad \forall k \in K$$

Total Battery Power



Timed Deliveries

Problem:

- Requests have an associated delivery time.

Solution:

- Impose arrival and departure times at each delivery site.
- Implicitly replace subtour elimination constraints.

$$a_i - d_i + T_s \leq 0, \qquad \forall i \in (V - K)$$

$$d_i - a_j + t_{ij} \leq Z\left(1 - \sum_{k \in K} x_{ij}^k\right), \qquad \forall (i,j) \in E, j \neq \omega$$

$$T_{start} \leq a_i \leq T_{start} + t_{bound}, \qquad \forall i \in (V - K)$$

$$T_{start} \leq d_i \leq T_{start} + t_{bound}, \qquad \forall i \in V$$

Arrival and Departure times

$$d_k - Z(1 - x_{k\omega}^k) \leq T_{start}, \qquad \forall k \in K$$

Preventing Stagnation

Soft Delivery Timings

Problem:

- Hard delivery timings make problem infeasible.

Solution:

- Impose soft penalties for delivery time deviations.
- Permit skipping deliveries.

Objective Fn: $\min_{x,a,d} \{f_{time}(d) + \lambda f_{travel}(x,a,d)\}$

where

$$f_{time}(d) = \sum_{i \in (V-K)} p_i(d_i - \tau_i + T_{A,i})^2$$

$$f_{travel}(x,a,d) = \sum_{k \in K} \left(\sum_{\substack{(i,j)\in E \\ j \neq \omega}} x_{ij}^k (a_j - d_i) + \sum_{\substack{(i,j)\in E \\ j = \omega}} x_{ij}^k t_{ij} \right)$$

$$\sum_{k \in K} \sum_{j \in V \cup \{\omega\}} x_{ij}^k \leq 1, \quad \forall i \in V$$

Relaxed Deliveries

$$T_{start} + t_{bound}\left(1 - \sum_{k \in K} \sum_{j \in V \cup \{\omega\}} x_{ij}^k \right) \leq d_i \quad \forall i \in V$$

Penalty for missing delivery

Full MIQP Formulation

Objective: $\min_{x,a,d} \{f_{time}(d) + \lambda f_{travel}(x,a,d)\}$

where

$$f_{time}(d) = \sum_{i \in (V-K)} p_i (d_i - \tau_i + T_{A,i})^2$$

$$f_{travel}(x,a,d) = \sum_{k \in K} \left(\sum_{\substack{(i,j)\in E \\ j \neq \omega}} x_{ij}^k (a_j - d_i) + \sum_{\substack{(i,j)\in E \\ j = \omega}} x_{ij}^k t_{ij} \right)$$

subject to the constraints:

Path Continuity:

$$\sum_{k \in K} \sum_{j \in V \cup \{\omega\}} x_{ij}^k \leq 1, \qquad \forall i \in V$$

$$x_{\alpha k}^k = 1, \qquad \forall k \in K$$

$$\sum_{i \in V} x_{i\omega}^k = 1, \qquad \forall k \in K$$

$$\sum_{i \in \{\alpha\} \cup V} x_{ih}^k = \sum_{j \in (V-K) \cup \{\omega\}} x_{hj}^k, \qquad \forall h \in V, k \in K$$

Time Flow:

$$a_i - d_i + T_s \leq 0, \qquad \forall i \in (V-K)$$

$$d_i - a_j + t_{ij} \leq Z\left(1 - \sum_{k \in K} x_{ij}^k\right), \qquad \forall (i,j) \in E, j \neq \omega$$

$$T_{start} \leq a_i \leq T_{start} + t_{bound}, \qquad \forall i \in (V-K)$$

$$T_{start} \leq d_i \leq T_{start} + t_{bound}, \qquad \forall i \in V$$

$$T_{start} + t_{bound}\left(1 - \sum_{k \in K} \sum_{j \in V \cup \{\omega\}} x_{ij}^k\right) \leq d_i, \qquad \forall i \in V$$

$$d_k - Z(1 - x_{k\omega}^k) \leq T_{start}, \qquad \forall k \in K$$

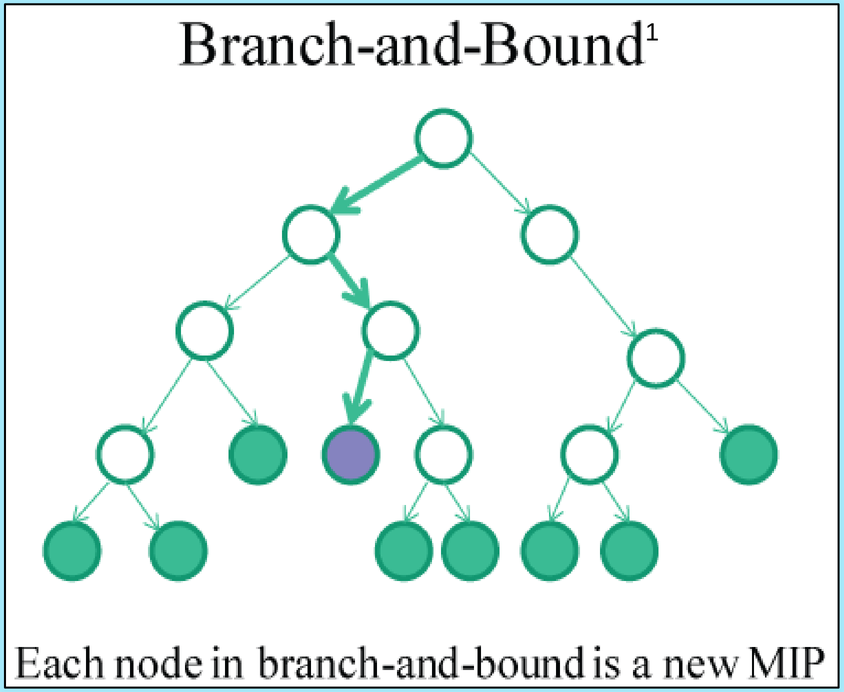
Capacity:

$$\sum_{(i,j)\in E} x_{ij}^k - 1 \leq C^k - c^k, \qquad \forall k \in K$$

$$\sum_{(i,j)\in E} x_{ij}^k B_r^k(t_{ij} v) \leq B^k, \qquad \forall k \in K$$

Solving the Scheduling Problem

- Always feasible! (Admits at least one solution: all ${x_{ij}^{k}}'s = 0$)
- MIQP (with non-PSD Hessian matrix) is NP-hard.
- Solution technique used:

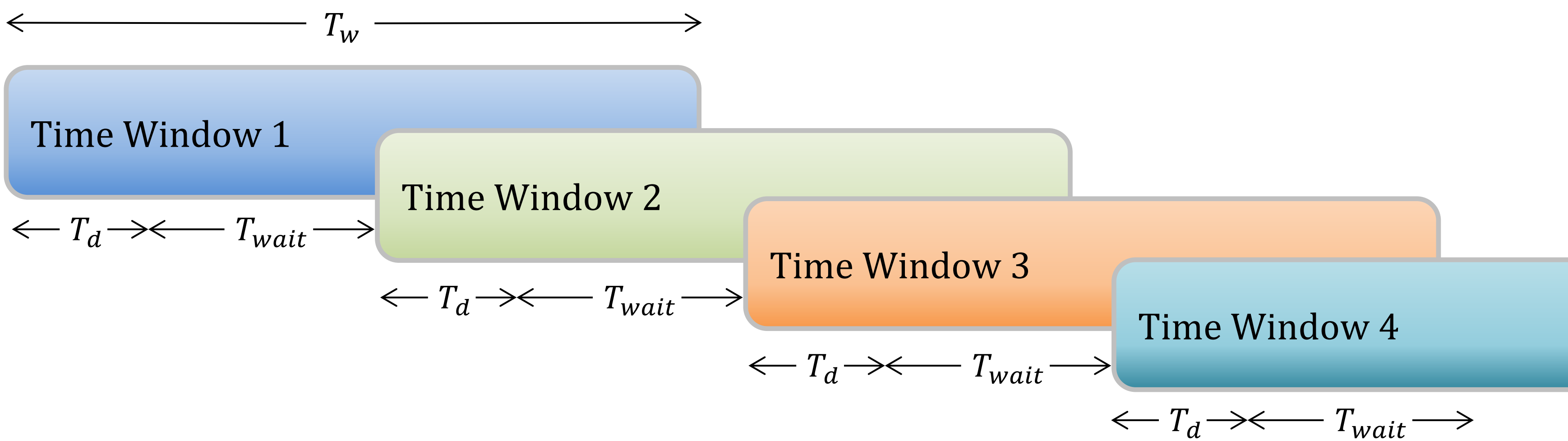


Solver: SCIP[2]
(Solving Constrained Integer Problems)
from
OPTI toolbox

1) Image taken from URL: http://www.gurobi.com/resources/getting-started/mip-basics
2) T. Achterberg, "SCIP: Solving constraint integer programs," *Mathematical Programming Computation* 1(1), pp. 1-41, 2009

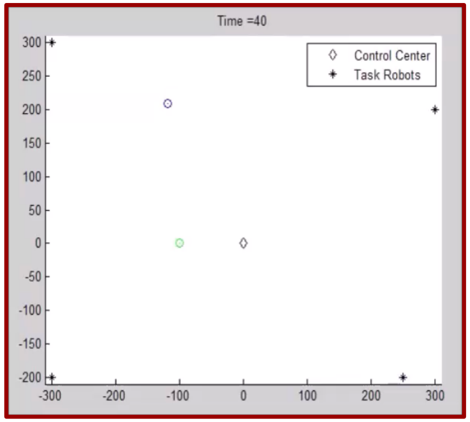
Online System: Time Windows

- Complete list of requests not available a priori.
- Finite horizon → Time Window scheduling.
- Allows dynamic rescheduling.

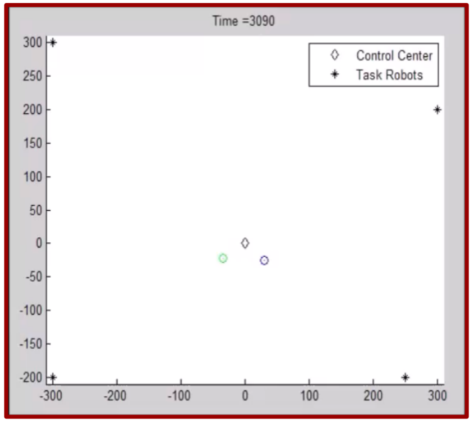


Results

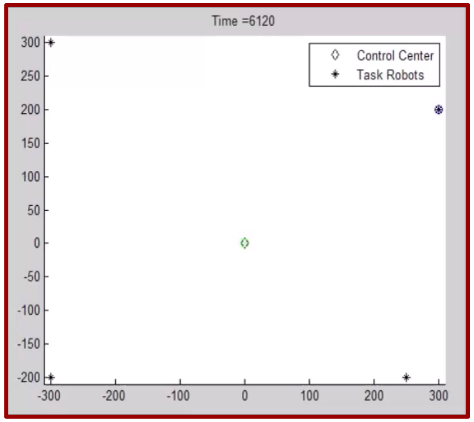
Window1: [40, 3080] s



Window2: [3080, 6120] s



Window3: [6120, 9160] s



Delivery robots

(based on AscTec Hummingbirds)

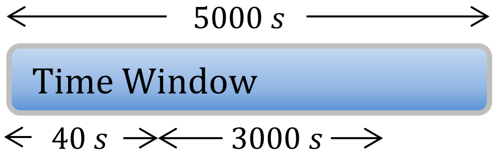
- v = 2 m/s
- Battery life: 1800 m (approx)
- B^k_r = 1/1800 units/m
- Max capacity: Blue=3, Green=2

Task robots

(ground robots)

- Battery life: About 1-2 hr

Scheduling parameters:



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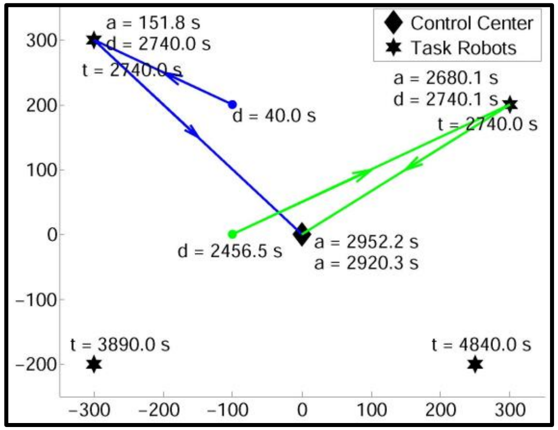

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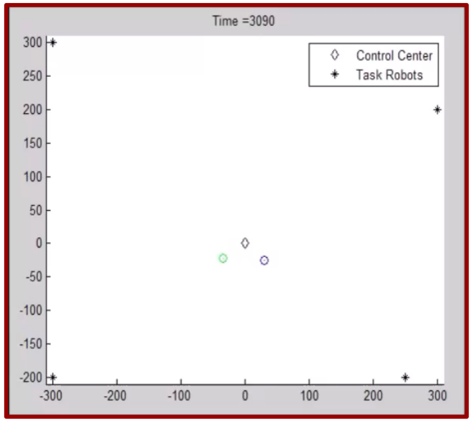

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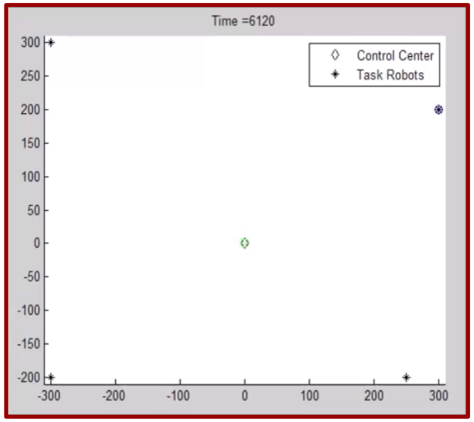

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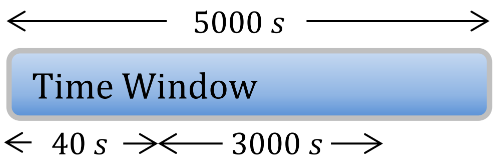

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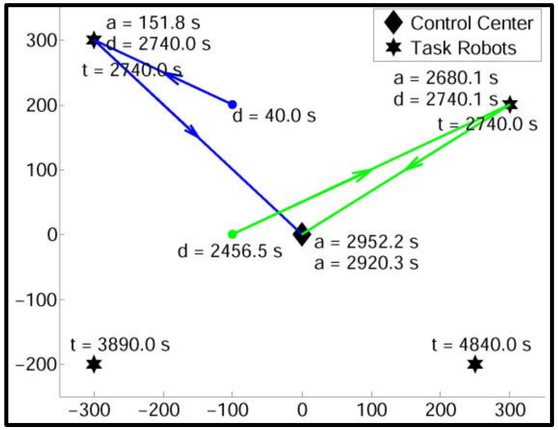

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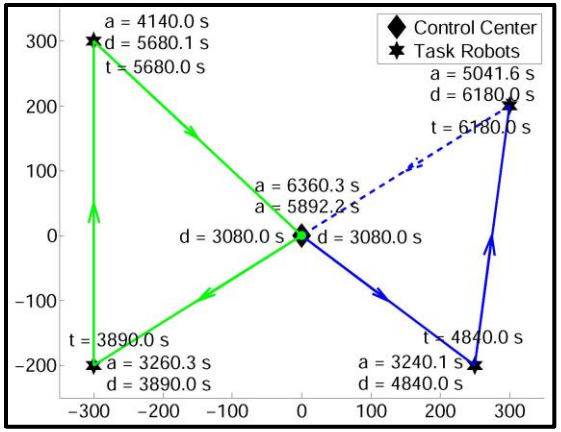

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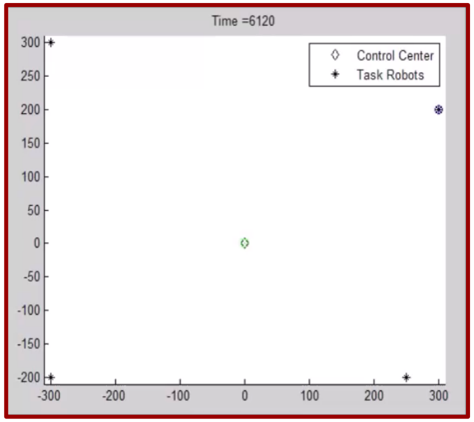

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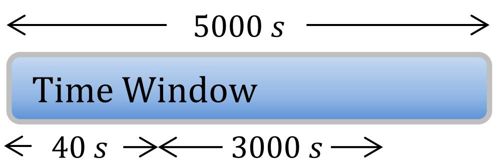

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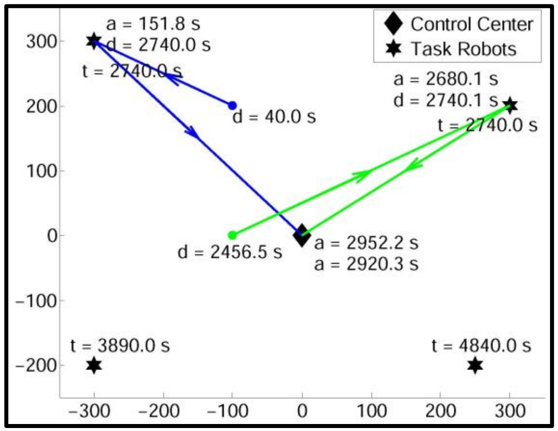

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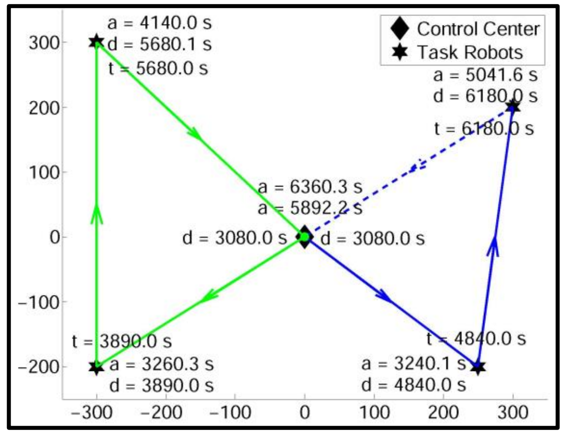

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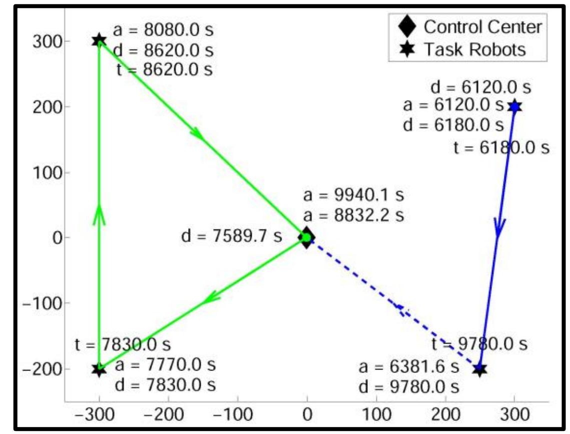

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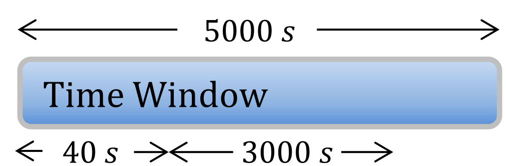
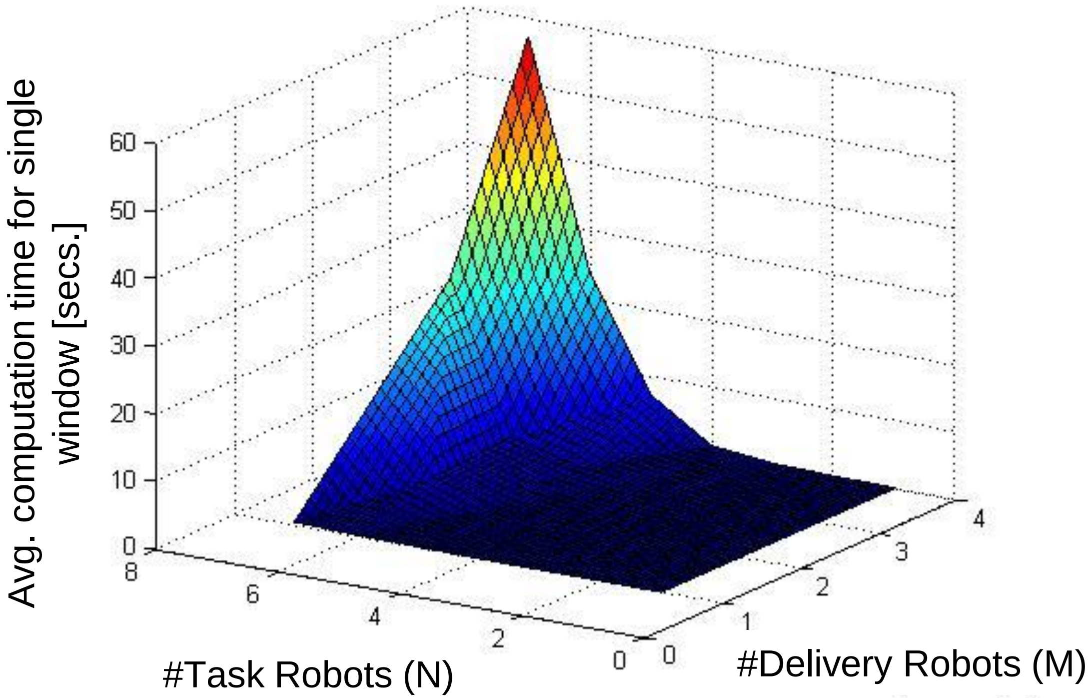


Time Complexity



- 500 single window trials with random number of robots, locations, delivery timings etc.
- Computation times averaged over instances with same values of M and N.
- Exponential growth – Useful for small groups of robots.

Contributions

- Solved the resource delivery problem with timed requests optimally.
- Problem formulation always has a feasible solution.
- Relaxed scheduling permitted when there is lack of resources or delivery robots.
- Enable dynamic re-routing of delivery robots enroute.
- Impose relative priorities when all task robots are not equally important.

Future Work

- **Tradeoff**: Approximate solution vs. faster computation.
- Decentralized planning.
- Removing synchronous time windows, to make planning asynchronous.

Thank you

Questions?